Volatility, Financial Markets and The Minority Game

Ryan Walch

Why is Volatility Important?

- High Frequency Trading has sped up financial markets
- As a result markets are more complex then ever. This has lead to worries about volatility.
- This can be seen in the Flash Crash of 2010
- Need a model to study volatility

Empirical Results of HFT

- HFT is between 27%(Benos and Sagade) and 73% (Hendershott) of all trades
- Two types
  - Passive Liquidity Supplying Market Makers
  - Aggressive Liquidity Demanders
- HFT Decrease volatility, increase liquidity help bring prices closer to the theoretical correct price.

The Minority Game

- 2k+1 Agents
- Two actions 1 and -1
- Agents are rewarded if they are on the minority side
- No a-priori best action
  - Agents will have difficulty forming correct beliefs
  - Agents act inductively
- Bounded Rationality
  - Agents must adjust their actions

The Minority Game and Volatility

- Social Optimum Occurs when the variance around the mean is minimized.
- Many actions in financial markets depend on a degree of balance.

Nash Equilibria

- If exactly k+1 agents are playing one action then that configuration is a Nash equilibrium
- If every agents chooses their action with even probability then the action profile is a Nash equilibrium.
- An infinite number of other Nash Equilibrium with some agents playing pure strategies and other agents are playing mixed strategies

Learning in the Minority Game

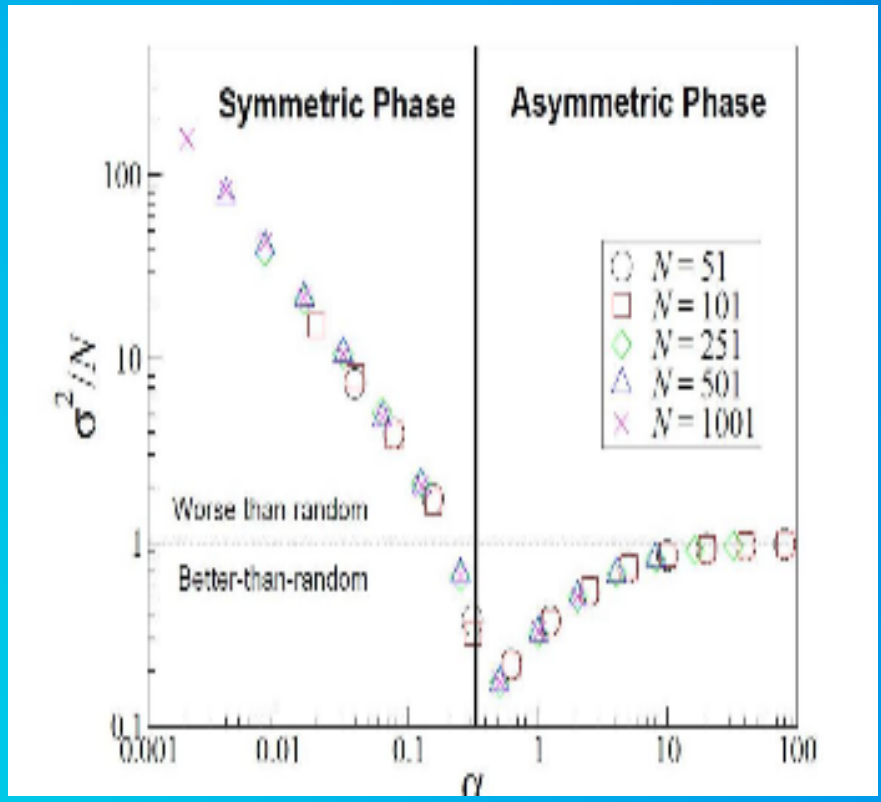
- A strategy is a rule which prescribes an action for a given history.
- Agents have a memory of length M. Agents use the history of the past M rounds to determine their actions
- Agents randomly draw s strategies.
- Each strategy has a score which increases when the agent does well using it
- This learning model is hard to analyze analytically

An example of a Strategy

History	Action
1,1,1	1
1,1,-1	1
1,-1,1	-1
1, -1, -1	1
-1, 1, 1	-1
-1, 1, -1	1
-1, -1, 1	1
-1,-1,-1	-1

Numerical Simulations and Alpha

- Alpha = $2^m/2k+1$ is the most important parameter in the minority game
- Intuitively Alpha is a measure of the crowdedness of the state space
- Crowds/anti-crowds



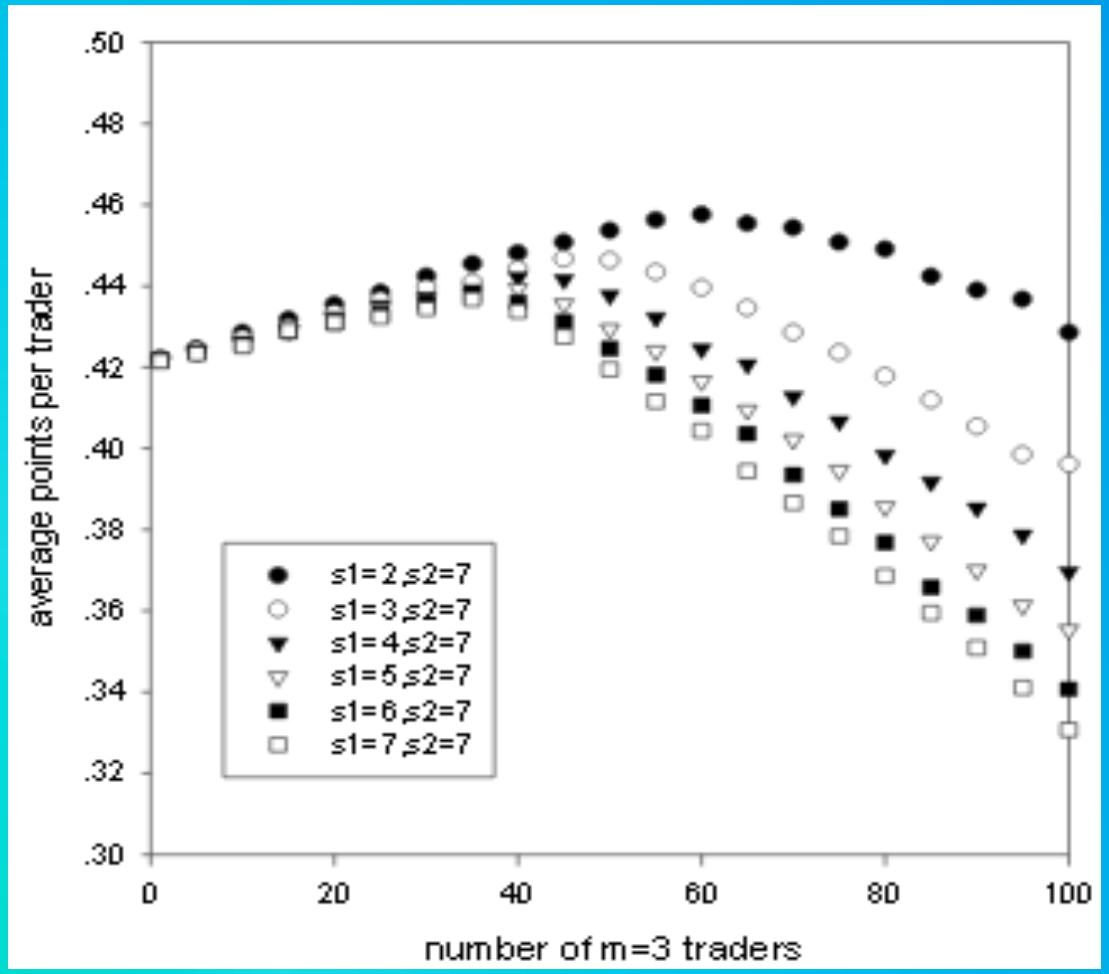
Other Learning Models

- Fictitious Play
  - Normal fictitious play will converge to the equal probability mixed strategy but will alternate
  - Stochastic fictitious play will converge as below
- Replicator Dynamics/Reinforcement Learning
  - Actions which result in good consequences are used more and actions which result in bad consequences are used less
  - Pure Strategy NE stable mixed Strategy NE Unstable

HFT as Longer Memory

- One way to model HFT is to allow some traders to have longer memory lengths
- This would represent the ability of HFT to process information more quickly then traditional investors
- Johnson et al. numerically simulated this case and found that mixed populations resulted in less volatility.

Johnson et al. (1999)



HFT as last mover advantage

- Another way to change the minority game into an extensive form game.
- If the proportion of HFT is small then if HFT moved last some volatility could be eliminated around the equilibrium

Conclusion

- Strategic Heterogeneity is Important
- Strategic Heterogeneity provides an additional way HFT can decrease volatility.
- Other aspects of HFT such as positional externalities, adverse selection and fairness need to be considered